



**A Frequency and Event Duration Analysis  
Of the State of New Hampshire's  
Proposed Instream Flow Rules**

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**Prepared For:**

**The New Hampshire Department of Environmental Services**

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## 1. Introduction

The purpose of this study is to examine the New Hampshire Department of Environmental Services' proposed In-stream Flow Rules, or IFR. Before the proposed IFR can move forward to the state Legislature, the potential economic impacts to the regulated community must be assessed. To do so, NH DES developed an agreement with Hydrologic Services, Inc. to perform an in-depth analysis of historic U.S. Geological Survey streamflow time series data and statistically summarize how the target, regulated community might have had to behave/comply with the IFR. Other parties who are responsible for performing the economic impact assessment analysis will use these statistics.

Using 34 U.S. Geological Survey stream gage sites, selected by the NH DES, three “random variables” are examined and their statistics are presented in this report. The first random variable is streamflow. The streamflow is summarized by special statistics, referred to herein as *quantiles*. A quantile is some magnitude of streamflow that is equaled or exceeded with some probability. Notation wise, a quantile is frequently written as “ $Q_p$ ,” where the subscript “p” is may be written as a *percentile*, the flow rate, Q, exceedance probability multiplied times 100. The closer the percentile is to 100, the smaller the streamflow rate, in other words,  $Q_{60} > Q_{90}$ .

The proposed IFR invokes various conservation and shut-off orders depending upon the specific quantile.  $Q_{60}$ , a flow rate that is equaled or exceeded approximately 60 percent of the time during the period-of-record sampled is a conservation flow rate.  $Q_{80}$  is a more conservative flow rate in that relative to  $Q_{60}$  flow conditions, the regulated community will need to conserve more water.  $Q_{90}$  is the proposed shut-off threshold quantile. To further assist NH DES, and make for a more robust study, additional quantiles are estimated. These include  $Q_{70}$ ,  $Q_{95}$  and  $Q_{98}$  and  $Q_{7,10}$ .  $Q_{7,10}$  is a special annual, low streamflow statistic used for many years to assist in the design of wastewater treatment plant process trains. Following a review of the present study, NH DES may chose to alter the IFR by using different quantiles and the attendant results will assist in those deliberations.

The second random variable and subsequent set of statistics developed during this study and described in this report is the number of regulatory events each year. Sampling the stream gage time series data relative to a specific IFR quantile by carefully following the regulatory event definition develops this random variable. During a long period-of-record, there may be wet years/seasons during which the IFR would not have been triggered. Conversely, during some dry years/seasons,

the IFR might be triggered into place many times. For each quantile, the average or mean, standard deviation, minimum, maximum, and 95% confidence interval of the mean number of IFR events per year are determined by calendar year and IFR season. For the entire period-of-record of streamflow data, only the mean number of events per year can be quantified.

The third random variable and subsequent set of statistics developed and reported on in this study focuses on the duration characteristics of a regulatory event. Sampling the stream gage time series data relative to a specific IFR quantile in accordance with the regulatory event definition develops this random variable. A short duration dry spell following by a rainstorm may result in a short duration IFR event that lasts only four days, the minimum event duration. A sustained dry spell might result in an IFR event that lasts for several months. For each quantile, the average or mean, standard deviation, minimum, maximum, and 95% confidence interval of the mean event duration in days by calendar year, IFR season and for the entire period-of-record, are presented.

Subsequent chapters describe how the quantile, number of regulatory events per year and the event duration statistics are estimated. Following that presentation is some preliminary analysis designed to assist NH DES and others with their analysis of the results presented herein. Appendix I lists the 34 U.S. Geological Survey stream gages used in this study. Appendix II is a presentation of the various estimated streamflow quantiles for each of the 34 stream gages. Appendix III is a presentation of the statistics of the number of events per year, by quantile and by stream gage. Appendix IV is a presentation of the statistics of the event duration in days, by quantile and by stream gage. Appendix V is an example time series for one USGS streamgauge period-of-record, Q<sub>90</sub> regulatory event.

## 2. Estimating the Streamflow Quantiles

The  $Q_{60}$ ,  $Q_{70}$ ,  $Q_{80}$ ,  $Q_{90}$ ,  $Q_{95}$  and  $Q_{98}$  seasonal threshold streamflow quantiles are estimated using a technique described by Vogel and Fennessey (1994). Using the longest available continuous calendar year (Jan 1 through Dec. 31) period-of-record for the 34 U.S. Geological Survey stream gages listed in Appendix I, the data are parsed into four separate populations. Population 1 comprises all streamflow records for days between January 1 and March 15 (winter). Population 2 comprises all streamflow records for days between March 16 and May 31 (spring). Population 3 comprises all streamflow records for days between June 1 and October 31 (summer). Population 4 consists of all streamflow for days between November 1 and December 31 (autumn).

Let  $n$  equal the number of daily observations from one of these four populations. Let the  $i$ th member of one of these four populations be described as  $q_i$  where  $i=1,n$ . If the streamflow observations of this population are rank-ordered then the result is the set of order statistics,  $q_{(i)}$ , where  $i = 1,n$ . Here  $q_{(1)}$  equals the largest observed value among all  $q_i$  and  $q_{(n)}$  equals the smallest observed value among all  $q_i$ . In other words, the daily data belonging to a specific population is arranged from largest to smallest.

Using the notation provided by Vogel and Fennessey (1994), each quantile is estimated using the weighted estimator first presented by Parzen (1979), shown below as Equation (1).

$$Q_p = (1-p)q_{(i)} + pq_{(i+1)} \quad (1)$$

where  $i=[(n+1)p]$  and  $\theta = [(n+1)p-i]$  and  $p$  equals the exceedance probability,  $p=P[Q \geq q]$ . The quantile estimator  $Q_p$  is undefined for values of  $p$  that lead to  $i=[(n+1)p]=0$ .

The  $Q_{7,10}$  is estimated for each of the 34 stream gauges. This statistic is provided for a point of comparison and is not presently being considered for use as a regulatory threshold simply because it is an annual, not a seasonal, statistic. Each gage's complete period-of-record is parsed into annual sub-sets of  $n_{day}=365$  or  $366$  calendar year daily observations. The annual seven-day low-flow for the  $i$ th of  $n_{years}$  of record is determined by Equation (2), and shown below. For  $j=1,n_{day}-6$

$$Q_{7,i} = MIN \frac{1}{7} \sum_j^{j+6} Q_j \quad (2)$$

where  $Q_{7,i}$  is the  $i$ th year's, smallest seven consecutive days of streamflow. These  $n$ year annual values (for example, thirty numbers for a thirty year record) are rank ordered thereby becoming order statistics. Equation (1) is then used to estimate the  $T=10$  year lowflow event with corresponding exceedance probability  $p$  equal to 0.9.

### 3. The Instream Flow Rule Regulatory Event

In order to be able to develop a sample population of both the number of events per year variable and the event duration variable, a concise definition of that which constitutes a Regulatory Event is necessary. After lengthy collaboration with NH DES program staff, a final definition was developed. The final NH DES definition of a regulatory event, which is different from the version found in the original agreement between NH DES and HYSR, Inc., is provided below.

#### DESCRIPTION OF A “REGULATORY EVENT”

##### DEFINITIONS

**Regulatory seasons** are: winter – January 1 (12:01 AM) to March 15 (11:59 PM);

spring – March 16 to May 31; summer – June 1 to October 31; autumn – November 1 to December 31.

A “**Decision Day**” occurs when the commissioner decides, after reviewing streamflow data for previous days, to issue or lift an order. Streamflows measured on the “Decision Day “ are not used in the decision made on that day.

A “**Trigger Flow**” is an average daily streamflow standard that is compared to average daily streamflow to determine whether to issue or lift an order. Trigger flows are determined separately for each season and each watershed. For example, the summer Q60 (the average daily summer streamflow value that is equaled or exceeded 60% of the time) is a trigger flow.

An “**Order**” is a notice issued by the commissioner to restrict or cease consumptive water use, based on evaluation of average daily streamflow relative to a trigger flow. An order goes into effect at 12:01 AM on the day after a Decision Day. An orders expires at 11:59 PM on the Decision Day. Orders last for 10 days, unless conditions for earlier expiration are met.

A “**Regulatory Event**” or “**Event**” is a continuous sequence of days during which orders associated with a particular trigger flow are in effect. As long as orders are continuously in effect they count as a single event. Events associated with different trigger flows are different events. [For counting purposes events may be seasonal, annual or period-of-record events. A new seasonal event begins following a decision day after a new regulatory season has begun, even though the sequence of days during which the orders are in effect may be continuous. For annual event counting, a new event occurs following a decision day after a new regulatory season begins, even though the sequence of days during which the orders are in effect may be continuous. Period-of-record events are counted whenever a Decision Day results in a new order being issued]

## **REGULATORY EVENT DESCRIPTION**

Average daily streamflows from gage data are assessed daily against each trigger flow. An event begins concurrent with the issuance of a reduction/cessation order at 12:01 AM on the day after a Decision Day when average daily streamflow has been less than the trigger flow for the Decision Day during each of the prior four days. Streamflow is always assessed using the trigger flows for the current Decision Day. For example, for Decision Day January 1, if average daily streamflow has been less than the winter trigger flow associated with Q60 for each of December 31, 30, 29, 28 then an order is issued on January 1 that causes a winter Q60 event to begin at 12:01 AM on January 2.

A new order is issued at 12:01 AM on the 11<sup>th</sup> day after the previous order was issued if streamflow has been less than the trigger flow for the Decision Day during each of the four previous days. If this condition is not met, the order expires at 11:59 on the Decision Day and the event ends. In the example above, if streamflow has been less than the winter Q60 trigger for each of January 10, 9, 8, 7 then an order is issued on January 11 that causes a subsequent winter Q60 order to begin on January 12. This is considered a continuation of the event.

In addition to the criteria for order expiration above, a reduction/cessation order expires at 11:59 PM on a Decision Day when average daily streamflow for each of the four previous days has been greater than 1.5 times the trigger flow for the Decision Day. The ending of an order with a greater trigger flow also ends all orders with lesser trigger flows. Orders issued in one season continue into the next season. An order in the new season is not issued until an order issued in the previous season has expired. An event in the new season begins when an order is issued using the new season's trigger flow.

For concurrent orders, the most restrictive consumptive use limitations of any order in effect apply.

An example regulatory event time series data record is presented in Appendix V. This example was developed using the regulatory event definition relative to the HYSR, Inc. estimated Q90 seasonal quantile, developed especially for the period-of-record sub-set (versus annual or IFR season sub-sets). Similar data sets have been developed for each of the 34 streamgauges, Q<sub>60</sub>, Q<sub>70</sub>, Q<sub>80</sub>, Q<sub>90</sub>, Q<sub>95</sub> and Q<sub>98</sub> quantiles, by season, by year and for the period-of-record. These data sets, prepared by HYSR, Inc. will be maintained and distributed for use by others by the NH DES.

## **4. Estimating the Statistics of the Number of IFR Regulatory Events per Year**

Under Task 2, the mean and standard deviation of the number of regulatory events per calendar year, the number of regulatory event for each calendar season and the mean number of regulatory events per year based upon the entire period of record, are estimated using the above definition of a regulatory event. Additionally, a 95% confidence interval of the mean number of events per calendar year and the mean number of events per calendar season is estimated. Taking the number of

regulatory event per year as an example, let  $nevent_i$  equal the number of regulatory events, which would have occurred during the  $i$ th of  $nyear$  calendar years. Using the final definition of the regulatory event presented in the prior section of this report,  $nevent_i$ , is counted.

Given a period-of-record length of  $nyear$ , the mean number of regulatory events per calendar year,  $E[nevent]$ , are estimated using the Method of Moments as shown by Equation (3).

$$E[nevent] = \frac{1}{nyear} \sum_{i=1}^{nyear} nevent_i \quad (3)$$

Given a period-of-record length of  $nyear$ , the standard deviation of the number of regulatory events per year,  $SD[nevent]$ , is estimated using the Method of Moments, as shown by Equation (4).

$$SD[nevent] = \sqrt{\frac{1}{nyear - 1} \sum_{i=1}^{nyear} (nevent_i - E[nevent])^2} \quad (4)$$

The 95% confidence interval of the *true* mean number of regulatory events per calendar year,  $\mu_{nevent}$ , could be estimated by assuming that the distribution of the mean number of events per year is approximately normally distributed with the standard deviation of the mean number of events per calendar year (which is not equal to  $SD[nevent]$ ), unknown. With the range of continuous record length between 12 and 94 years, it could be assumed that the  $(1-\alpha)100\%$  confidence interval for  $\mu_{nevent}$  is described by Equation (5) as shown below.

$$\bar{x} - \frac{t_{\alpha/2} S}{\sqrt{n}} < \mu_{nevent} < \bar{x} + \frac{t_{\alpha/2} S}{\sqrt{n}} \quad (5)$$

where  $\bar{x}$  equals  $E[nevent]$ ;  $S$  equals  $SD[nevent]$ ;  $t_{\alpha/2}$  equals the value of the Student's  $t$  distribution with  $v=n-1$  degrees of freedom and  $n$  equals  $nyear$ . From standard statistical tables, for a  $\alpha=0.05$  significance level, and with  $v=12-1$  degrees of freedom,  $t_{0.025}=2.201$ . With  $v=\infty$ ,  $t_{0.025}=1.960$ . With both values so close to 2.0, this study instead assumes that the 95% confidence interval for the *true* mean number of events per year,  $\mu_{nevent}$ , is adequately described with  $t_{0.025}=2.0$  independent of  $nyear$ .

Given a stream gage period-of-record length of  $nyear$ , a total of  $K$  regulatory events occur. The mean number of regulatory events per year,  $E[kevents]$ , for the entire period of record, is estimated using the Method of Moments as shown by Equation (6).



$$E[kevent] = \frac{1}{nyear} \sum_{i=1}^k kevent_i \quad (6)$$

## 5. Estimating the Statistics of the Duration of an IFR Regulatory Event

Under Task 3, the mean and standard deviation of the duration of a regulatory event are estimated. Additionally, a 95% confidence interval of the *true* mean duration of a regulatory event is estimated. Let  $durat_i$  equal the duration in days of the  $i$ th of  $npor$  regulatory events that occurred during the entire period-of-record. Using the definition of the regulatory event presented earlier in this report, the sample population of  $durat_i$ ,  $i=1, npor$ , is developed.

Given  $npor$  events during the period-of-record, the mean duration of a regulatory event,  $E[durat]$ , is estimated using the Method of Moments as shown by Equation (7).

$$E[durat] = \frac{1}{npor} \sum_{i=1}^{npor} durat_i \quad (7)$$

Given  $npor$  regulatory events during the period-of-record, the standard deviation of the duration of a regulatory event,  $SD[durat]$ , is estimated using the Method of Moments as shown by Equation (8).

$$SD[durat] = \sqrt{\frac{1}{npor - 1} \sum_{i=1}^{nyear} (durat_i - E[durat])^2} \quad (8)$$

The 95% confidence interval of the *true* mean duration of a regulatory event,  $\mu_{durat}$ , could be estimated by assuming that the distribution of the mean event duration being approximately normally distributed with the standard deviation of the mean event duration,  $SD[durat]$ , unknown. One could assume that the  $(1-\alpha)100\%$  confidence interval for  $\mu_{durat}$  is described by Equation (9) and shown below.

$$\bar{x} - \frac{t_{\alpha/2} S}{\sqrt{n}} < m_{durat} < \bar{x} + \frac{t_{\alpha/2} S}{\sqrt{n}} \quad (9)$$

where  $\bar{x}$  equals  $E[durat]$ ;  $S$  equals  $SD[durat]$ ;  $t_{\alpha/2}$  equals the value of the Student's  $t$  distribution with  $v=n-1$  degrees of freedom and  $n$  equals  $npor$ . With  $npor$  found to be large, this study instead

assumes that the 95% confidence interval for the mean duration of a regulatory event is adequately described by letting  $t_{0.025}=2.0$ .

## **6. Preliminary Analysis and Discussion of the Results.**

A comprehensive analysis of the various numbers of regulatory events per year statistics is well beyond the scope of this study. The following will serve to illustrate the sheer magnitude of that task. Appendix III is a listing of the number of regulatory events per year statistics. There are 34 stream gages. Six quantiles are estimated for each gage. For each quantile, period-of-record, calendar year, winter IFR season, spring IFR season, summer IFR season and Fall IFR season statistics are estimated. For all but the period-of-record population, six statistics are estimated: the mean; the standard deviation; upper 95% confidence interval of the mean; the lower 95% confidence interval of the mean; the minimum number of regulatory events per year and the maximum number of regulatory events per year. As discussed earlier, only the mean number of regulatory events per year is estimated for the period-of-record. The reader might be interested to know that he or she will find 6,324 summary statistics in Appendix III.

A few observations are offered to help guide NH DES program staff and the Advisory Committee to perform a comprehensive assessment of the results. The approach offered herein might serve to help them devise an approach to reduce this work in a way that the economists will find tractable. Focusing solely on the mean (average) number of regulatory events per year statistic, found in Appendix III, all 34 (USGS gage sites) are plotted by quantile. Figure 6.1, upper plot, shows the distribution of this statistic for the period-of-record, the lower plot shows the distribution of this statistic for the calendar year (annual). Figure 6.2, upper plot, shows the distribution of this statistic for the winter IFR season, the lower plot shows the distribution of this statistics for the spring IFR season. Figure 6.3, upper plot, shows the distribution of this statistic for the summer IFR season, the lower plot shows the distribution of this statistics for the fall IFR season.

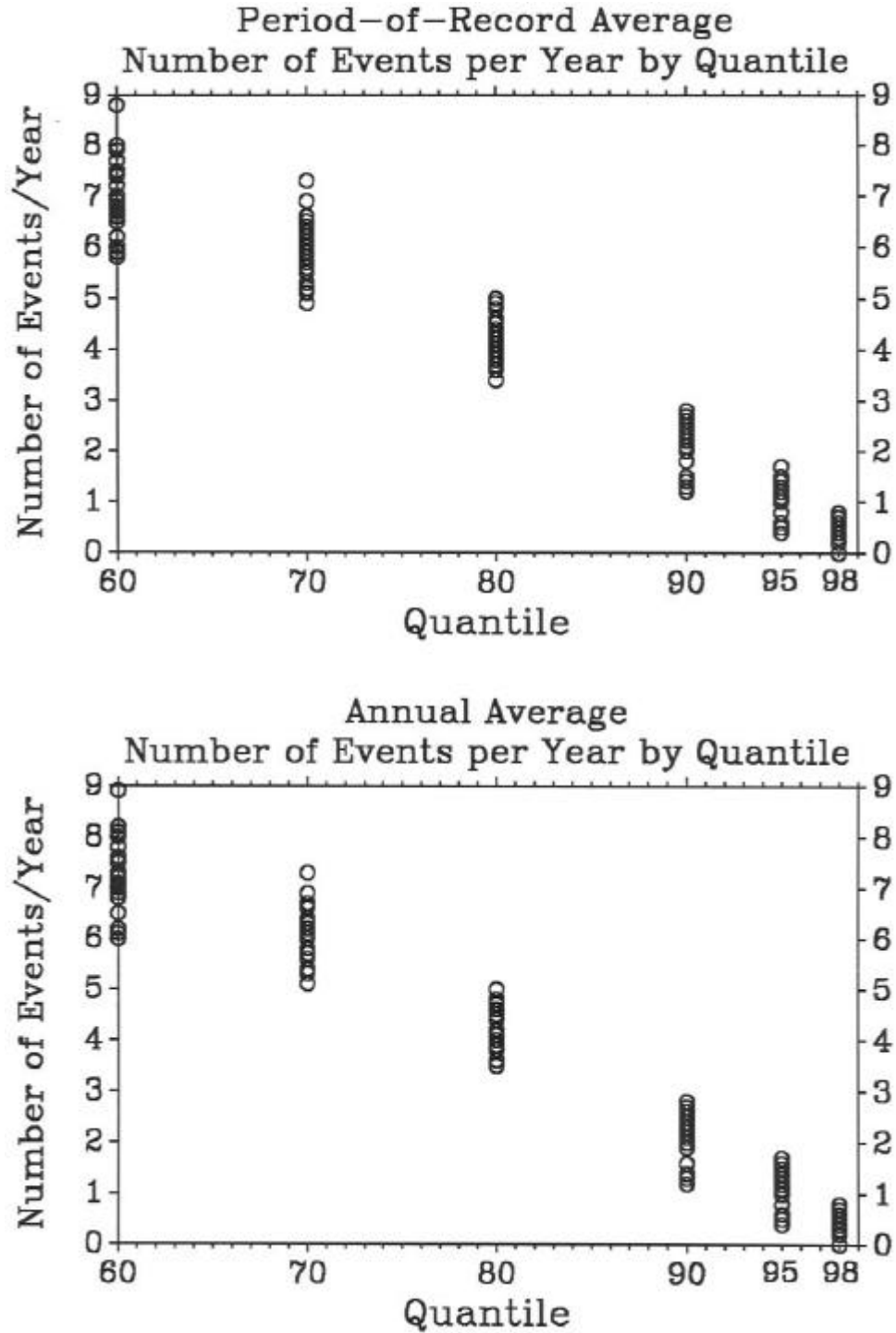


Figure 6.1 POR and Annual Average Number of IFR Events per Year

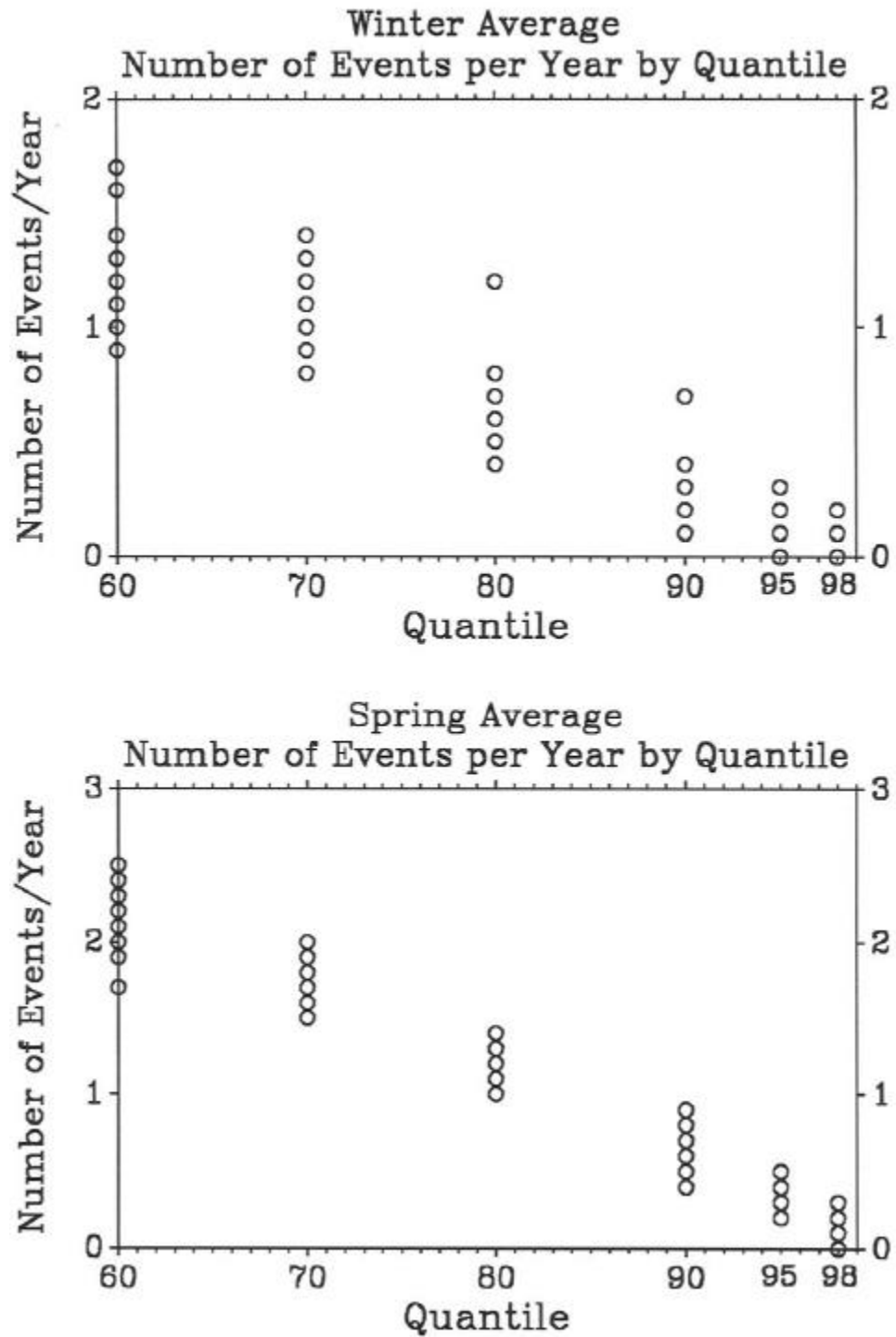


Figure 6.2 Winter and Summer Season Average Number of IFR Events per Year



An unexpected can be seen. Despite the high degree of regulation upstream from a number of the 34 stream gage sites, the broad difference in climate, geomorphology and especially the range in size of the watershed areas, the average number of regulatory events per year is remarkably uniform. For example, the regulated community can expect to be required to cease withdrawing water ( $Q_{90}$ ) for between 1 to 3 times per year, on average as suggested by Figure 6.1. The shut-off season will most likely be the summer, as suggested in Figure 6.3. The reader should remember, however, that the summer IFR season lasts five months.

Similarly, a comprehensive analysis of the various event duration statistics is well beyond the scope of this study. Appendix IV is a listing of the regulatory event duration statistics. There are 34 stream gages. Six quantiles are estimated for each gage. For each quantile, period-of-record, calendar year, winter IFR season, spring IFR season, summer IFR season and Fall IFR season statistics are estimated. Six statistics are estimated: the mean; the standard deviation; upper 95% confidence interval of the mean; the lower 95% confidence interval of the mean; the minimum number of regulatory events per year and the maximum number of regulatory events per year. The reader might be interested to know that he or she will find 7,344 summary statistics in Appendix IV.

Focusing solely on the mean (average) duration of a regulatory event statistic, found in Appendix IV, all 34 (USGS gage sites) are plotted by quantile. Figure 6.4, upper plot, shows the distribution of this statistic for the period-of-record, the lower plot shows the distribution of this statistic for the calendar year (annual). Figure 6.5, upper plot, shows the distribution of this statistic for the winter IFR season, the lower plot shows the distribution of this statistics for the spring IFR season. Figure 6.6, upper plot, shows the distribution of this statistic for the summer IFR season, the lower plot shows the distribution of this statistics for the fall IFR season.

An unexpected result can be seen. Again, despite the high degree of regulation upstream from a number of the 34 stream gage sites, the difference in climate, geomorphology and especially watershed area size, the average duration of a regulatory event is quite uniform. For example, the regulated community can expect to be required to cease withdrawing water ( $Q_{90}$ ) for between 10 to 18 days per event, on average, as suggested by Figure 6.4. The variation of this statistic among the 34 gage sites increases during the winter, for example, which range between 8 and 28 days per event, as seen in Figure 6.5.

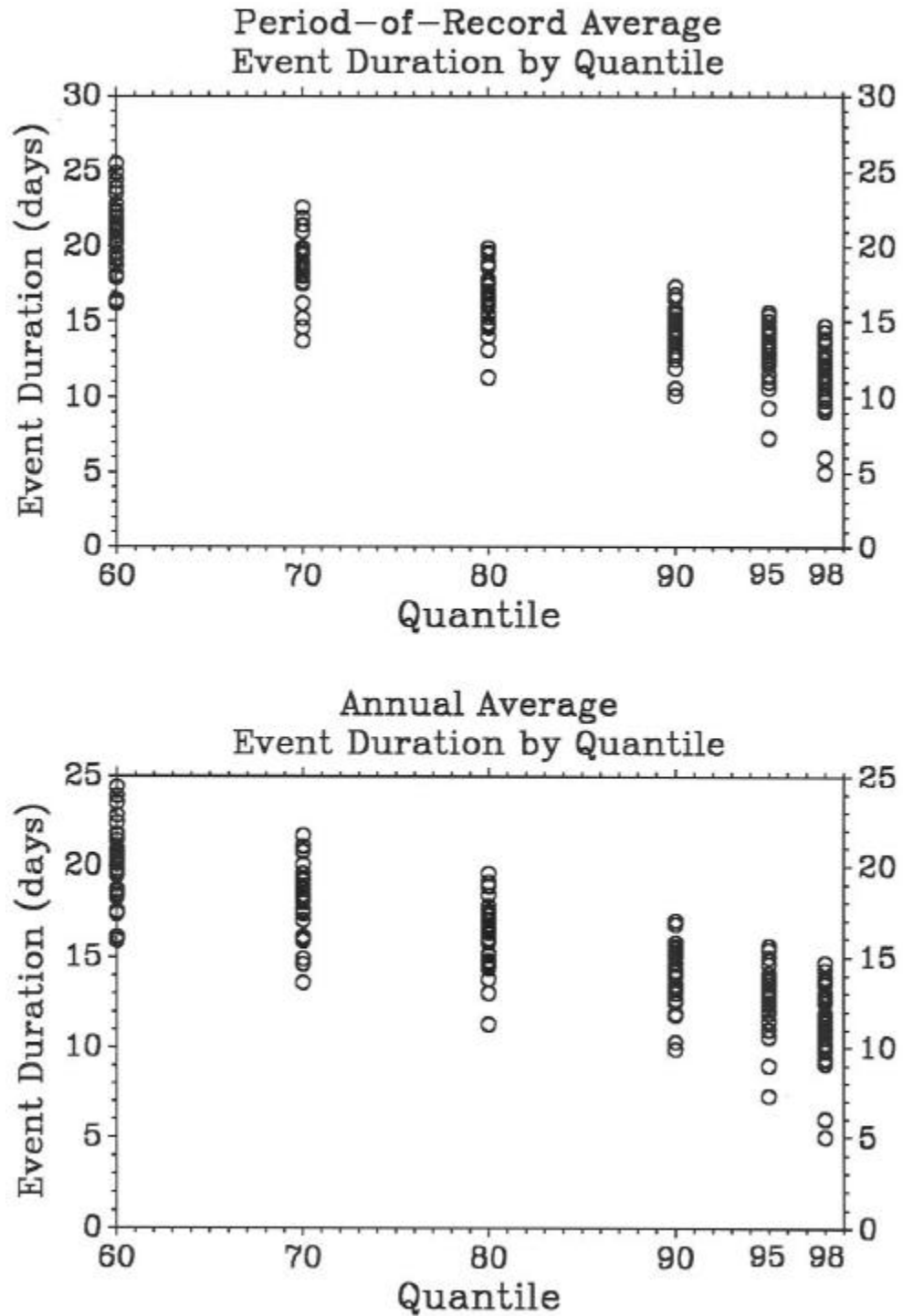


Figure 6.4 POR and Annual Average IFR Event Duration in Days

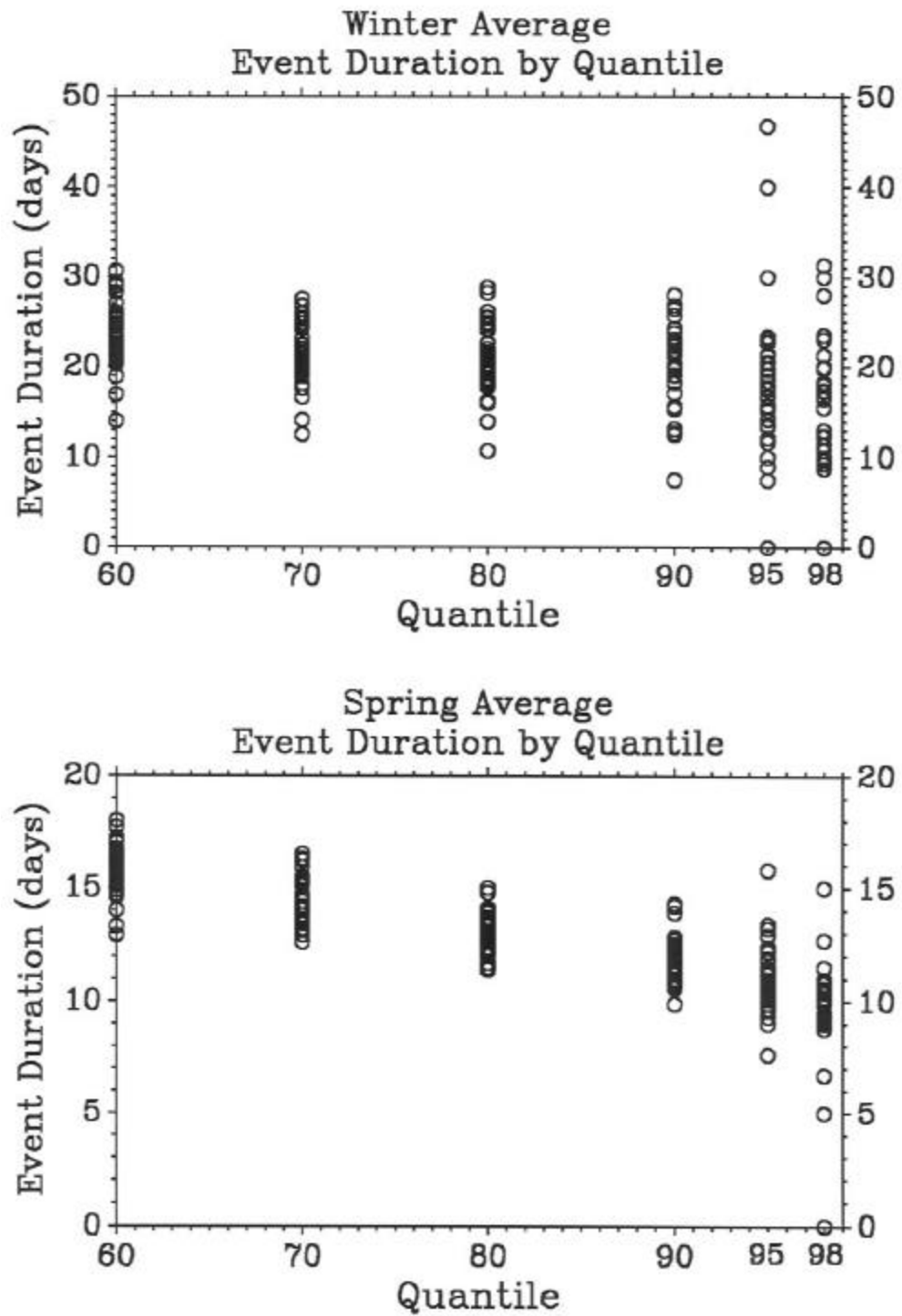


Figure 6.5 Winter and Spring Season Average IFR Event Duration in Days



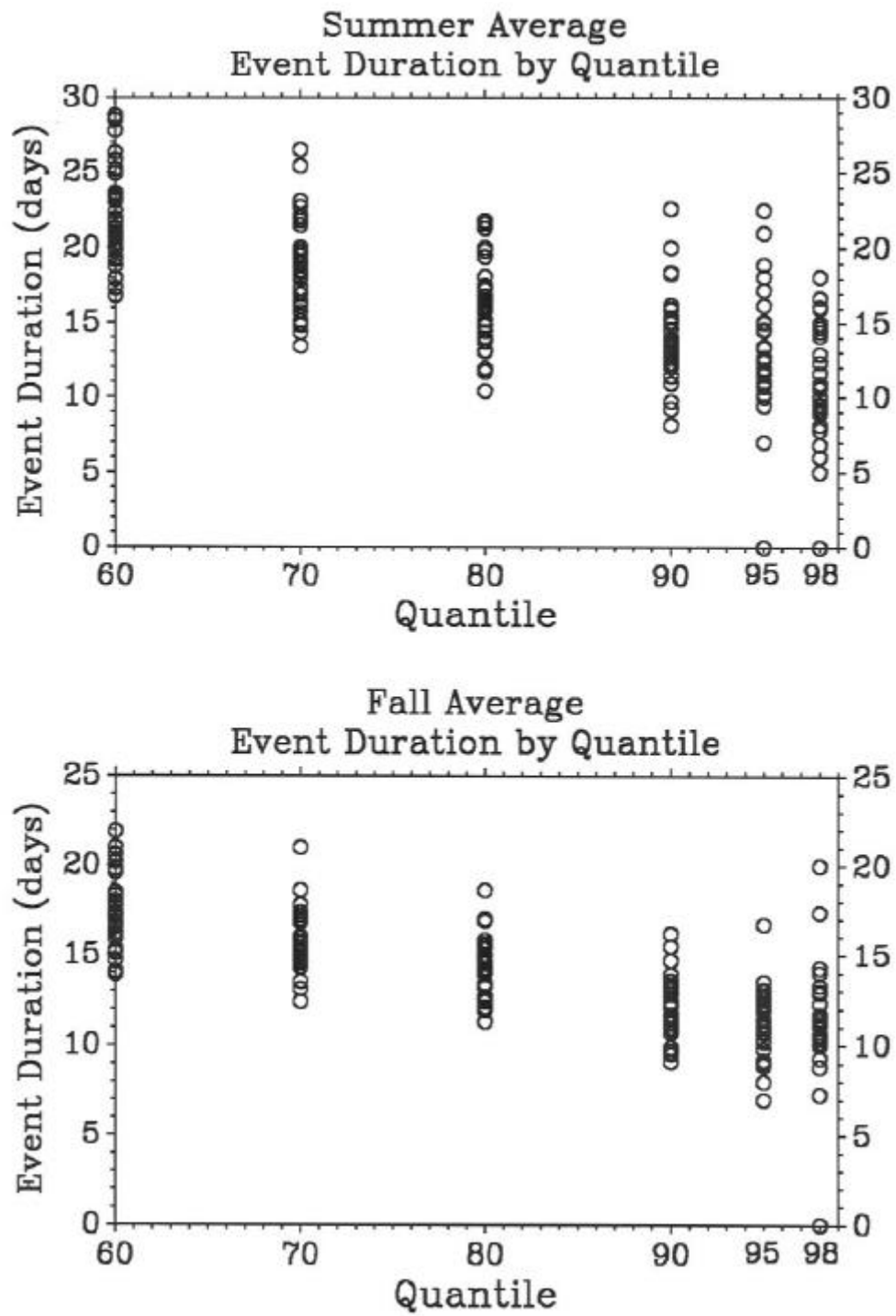


Figure 6.6 Summer and Fall Season Average IFR Event Duration in Days

While there are any number of ways in which these statistics might be summarized, one way is to take the average, median value or even the graphical mid-point of these statistics. Using the latter approach, using  $Q_{90}$  just for the sake of discussion, the regulated community can expect to cease withdrawing water twice a year on average. For each IFR event, the regulated community can typically expect to cease withdrawing water for 14 days. Simply multiplying these two independent statistics together derives one summary index that the economists and others might find useful. For example then, under the proposed Instream Flow Rules, the regulated community can expect to be required cease withdrawing water for one month every year, on average.

## **7. Off-Stream Storage Analysis and Event Duration Probability Analysis.**

Summarizing the population of regulatory events in terms of average values, standard deviations, confidence intervals, maximums and minimums are very useful ways to describe these events. For many among the regulated community, however, these statistics may not be sufficient to fulfill their needs. For example, from the preceding analyses, it's clear that the regulated community will be required to reduce and cease consumptive withdrawals fairly often. In order to avert critical supply shortfalls that could potentially adversely impact public health, manufacturing, etc., the construction of water storage facilities will probably be necessary. One aspect of this issue regards sizing the facility, which depends on the level of reliability or likelihood of service failure (risk) that the withdrawer is willing to accept. For a discussion of about reservoir reliability, see Vogel et al (1995) for example.

Using the event data files especially developed for NH DES, one may perform a storage facility reliability analysis. As mentioned earlier, Appendix V is one of the regulatory event time series data sets, which was then analyzed for both the event duration and the number of events per year statistics. This example data set is the period-of-record  $Q_{90}$  regulatory event time series data for Saco River near North Conway, N.H. streamgauge. For a particular streamgauge and quantile of interest and for a specific data set (by season, annual or period-of-record) the Weibull, or some alternative, plotting position may be used to assign an exceedance probability value to rank-ordered the event duration data (see Fennessey and Vogel, 1990). Example probability plots of the period-of-record  $Q_{60}$ ,  $Q_{80}$  and  $Q_{90}$  are presented respectively as Figure 6.7, Figure 6.8 and Figure 6.9.

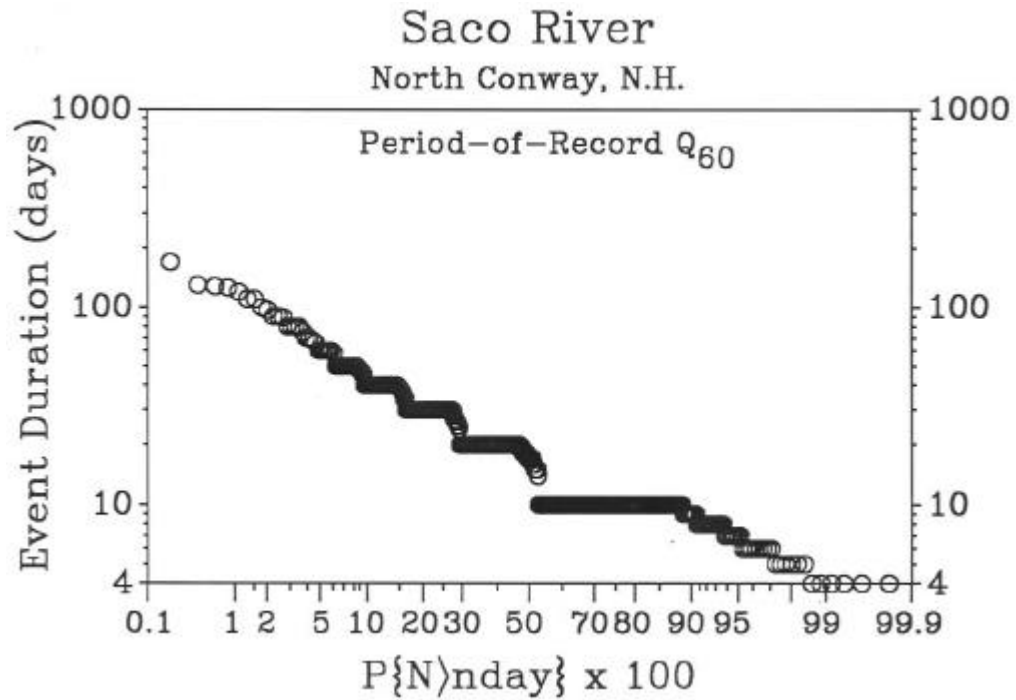


Figure 6.7  $Q_{60}$  Event Duration Probability Plot

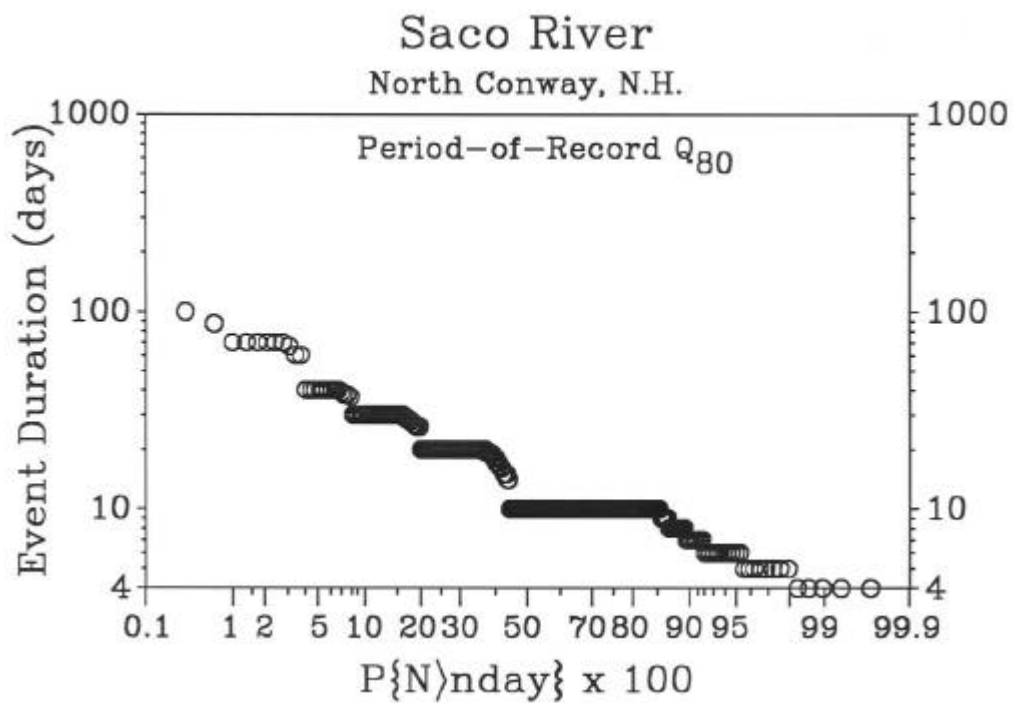
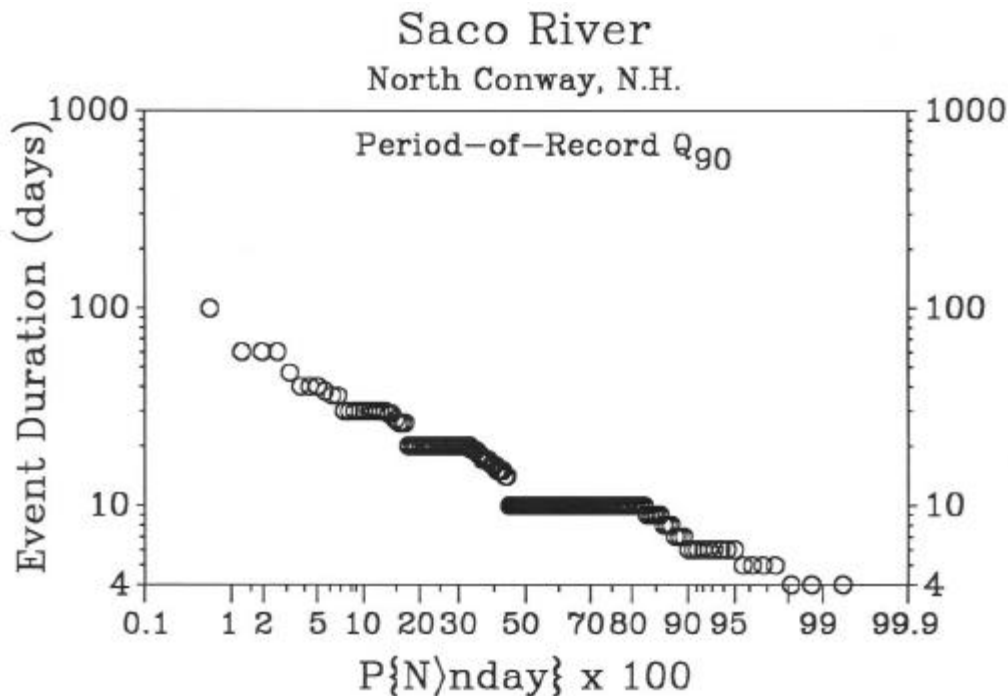


Figure 6.8  $Q_{80}$  Event Duration Probability Plot



**Figure 6.9 Q<sub>90</sub> Event Duration Probability Plot**

Using Figure 6.9 as an example, there is about 50% probability that a single regulatory event will last approximately 10 days or longer. Similarly, there is about a 5% probability that a single regulatory event will last approximately 40 days or longer. This means that for every twenty Q<sub>90</sub> regulatory events on average, a withdrawer on the Saco River near North Conway might expect one event to last for at least 40 days (and 40 nights.). Depending upon the degree of risk tolerance, one withdrawer might size a storage facility to provide for their complete needs for 10 days whereas another might size a storage facility to provide their complete needs for 40 days.

It's also important to understand what the probability plot does not say. It provides no information about the length of time between consecutive regulatory events. That information would be necessary to determine the likelihood of being able to refill the storage facility before the next event occurs. Fortunately, however, the special data sets prepared as a part of this study, which will be maintained and distributed by NH DES, will allow the competent analyst to perform a complete analysis.

## 8. Acknowledgements

Several individuals from the New Hampshire Department of Environmental Services were assisted the author with this study. In particular, the project manager Mr. Paul Currier, P.E. and the author were in constant communication, working closely on several key tasks, especially developing and clarifying the IFR event definition. He carefully checked and subsequently approved test output time series to ensure that the intermediate results conformed to the definition we developed. Mr. Ken Edwardson and Mr. Wayne Ives also assisted by reviewing test output. Mr. Rick Chormann and Mr. Jim MacCartney assisted with the development of a preliminary regulatory event definition. Finally, the author would like to thank both Mr. Currier and Mr. Ralph Abele of the U.S. Environmental Protection Agency Region 1 office in Boston, Massachusetts, for working together to secure a source of funding for the study.

## 9. References

- Fennessey, N.M. and R.M. Vogel, Regional Flow-duration Curves for Ungauged Sites in Massachusetts, ASCE J. of Water Resources Planning and Management, Vol. 116, No. 4, pp. 530-549, 1990.
- Parzen, E., (1979), “*Nonparametric statistical data modeling.*”, J. of American Statistical Association, 74(365), 105-122.
- Vogel, R.M. and N.M. Fennessey, (1994), “*Flow-duration curves. I: New interpretation and confidence intervals.*”, ASCE J. of Water Resources Planning and Management, 120 (4), 485-504.
- Vogel, R.M, N.M. Fennessey and R.A. Bolognese, *Storage-Reliability-Resilience-Yield Relations for Northeastern United States*, ASCE J. of Water Resources Planning and Management, Vol. 121, No. 5, pp. 265-274, 1995.